APPLICATION OF CHEBYSHEV’S THEOREM FOR 
ESTIMATING CO₂ EMISSIONS DUE TO 
OVERLOADING OF HEAVY DUTY DIESEL TRUCKS

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SUMMARY

The number of heavy duty diesel trucks (HDDT) loaded beyond the authorized limits is high. This overload causes additional CO₂ emissions that significantly increase the amount of pollutants emitted into the atmosphere. Measuring these emissions is complicated because it is required to know the truck’s operating conditions at all times. That is, it would be necessary to know truck’s speed, acceleration, load, and road roughness and slope. Certainly, recording these data for all vehicles at all times is impractical. In this work, a methodology for estimating these emissions from circulating overloaded and/or speeding trucks, based on the application of Chebyshev’s theorem, is presented. With it, only few experimental data are needed to validate a truck’s model. Once the truck model is validated, it is possible to simulate several operating conditions; including overloading and speeding. Also, using theoretical models, fuel consumption and CO₂ emissions are estimated within a certain statistical confidence level. In order to illustrate the proposed methodology, the problem of overloaded and speeding HDDT in México is analyzed. Simplified models for calculating tractive force, fuel efficiency and CO₂ emissions are used. It is estimated, within a 90% confidence level, that due to overloading and speeding, in México 60% more CO₂ is emitted compared to that produced if trucks were to operate within authorized loading and speed conditions.

KEYWORDS / Chebyshev Theorem / CO₂ / Diesel Trucks / Emissions / Overload / Speeding /

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theoretical formulation so as to obtain its fuel efficiency and hence its CO₂ emissions.

This paper proposes a methodology to estimate emission levels from overloaded trucks, based on the amount of fuel consumed. The procedure presented here, based on the application of Chebyshev’s theorem, allowed the use of different truck and emission models, according with the degree of statistical confidence to be achieved. Using known truck fuel efficiency, emission models and data, an application is herein developed for the Mexican case. An assertion can be made that there is an important increment in CO₂ emitted by an overloaded and speeding truck, compared to emissions produced by trucks operating within the legal weight and speed limits. Application of the proposed approach considers overload level, operating speed, type of road surface and road slope. Results are presented for five axle articulated and double trucks, which are the most representative trucks for this case.

Methodology to Estimate Fuel Consumption and CO₂ Emissions of Overloaded and Speeding Trucks

Approach taken for validating the theoretical formulation

Run a limited number of test trips (n₁) registering fuel consumption, type of terrain, road surface conditions, load levels and average travel speed.

Gather as much information as possible from trucking companies (n₂) related to their fuel consumption and trips operating conditions.

Choose a truck, fuel consumption and emission model, and considering truck’s and road characteristics from the test trips (n₃), calculate fuel efficiency and CO₂ emissions.

Using Chebyshev’s theorem, calculate Chebyshev’s intervals in order to validate within certain confidence statistical level, data coming from the chosen models.

Conduct the validation considering the Chebyshev’s intervals, the results produced by the chosen models and the experimental data collected.

Chebyshev’s theorem

Validation of the theoretically calculated data, based on that experimentally gathered, is carried out by constructing Chebyshev’s intervals (Kasmi er, 1998), assuming a predetermined confidence level. Coefficients of variability are also calculated for these intervals. Construction of Chebyshev’s intervals is carried out as follows.

The probability that any set of fuel efficiency values (X), expressed in km/l (with mean μ and variance σ²), takes values within the standard deviation of the mean value, is at least 1 - 1/k². That is:

\[ P\left(\frac{X - \mu}{\sigma} < k\sigma\right) \geq 1 - \frac{1}{k^2} \quad (1) \]

\[ P\left(\mu - k\sigma < X < \mu + k\sigma\right) \geq 1 - \frac{1}{k^2} \quad (2) \]

\[ P\left(\mu - k\sigma < X < \mu + k\sigma\right) = \sum_{k=0}^{\infty} \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{x^2}{2\sigma^2}} \quad (3) \]

Considering sample size n (n₁, n₂, n₃) and looking for a high degree of results reliability (Kasmi er, 1998), Chebyshev’s intervals can be calculated with a significance level α. Therefore, from expression α = 1/k² and a confidence level of 1-α, k, can be obtained from 1 - 1/k² = 1 - α for any k > 1. This means that the minimum proportion of fuel efficiency values that are within k times the standard deviation from the population mean is at least 1-α. In other words, it means that 1-α out of 100 samples of size n contain the population average fuel efficiency values.

The set of fuel efficiency values X satisfies the properties of a discrete random variable (Walpole, 1978; Daniel, 2011), as

\[ P(X = x) = f(x) \geq 0 \quad (a) \]

\[ \sum_{x} P(X = x) = 1 \quad (b) \]

X is a random variable with sampling distribution for a sample size n (Blalock, 1972; Daniel, 2008). Since the population mean and standard deviation are unknown, they are replaced by the sample mean and its standard deviation, calculated as

\[ \bar{x} = \frac{\sum x}{n} \quad (4) \]

\[ s_x = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}} \quad (5) \]

It is stressed that Chebyshev’s inequality is rarely used to set mean value confidence intervals (in our case fuel efficiency average for both data, experimental and calculated). However, in this case, this is the best method to apply given a population that is not normal and only small samples are available (Kasmi er, 1998).

From Eq. 3, Chebyshev’s intervals are set as

\[ \bar{x} \pm k \times \frac{s_x}{\sqrt{n}} \quad (6) \]

Fuel Consumption and CO₂ Emission

Although there are few recent estimates of the global fuel consumption and related CO₂ emissions of road transport (Halenka and Uherek, 2010), the effects of vehicle weight and road grade have not been studied in detail (Naguib, 2011). A majority of emission models include a number of vehicle characteristics, travelled mileage, and driving modes, but other important factors, such as vehicle weight, road grades, and weather effects, are not sufficiently addressed as they are difficult to predict...
or measure (Chunxia, 2005; Suzuki, 2011). The main types of emission models are speed-based, modal, and fuel-based models. The first two types are the most commonly used. In speed-based models, the main input is the average speed; simplicity and limited data needs are their main advantages, but they do not account for driver (speed fluctuations) and roadway influences. Examples of speed-based emissions models include EPA MOBILE (EPA, 2009; COPERT, 2009; EMFAC, 2007). Modal models are based on vehicle’s operating mode determined from roadway, driver, and traffic factors to calculate emissions.

In order to illustrate the methodology presented, the well-known relations involving road slope, rolling and aerodynamic resistances found in (Fitch, 1994), Chunxia and Seungju (2005) and Morales et al. (1995) are used for a truck model. For fuel consumption and CO₂ emissions estimation, the models proposed by Torres (1998) and by Zadek and Schulz (2010) are used.

### Tractive force

This model assumes there is a tractive force equal to the sum of forces opposing the truck’s movement. It is assumed that the truck travels at a constant speed, thus the tractive force \( F_t \) available at the wheels is:

\[
F_t = F_r + F_a + F_v
\]

where each force given by

\[
F = \frac{g \cdot m \cdot \sin(\theta)}{2}
\]

\[
F_r = \frac{d}{2} \times C_d \times A_f \times V^2
\]

\[
F_v = (C_g \times W + C_w \times W \times V) \times r
\]

Considering international units for expressing all forces in Newtons, the tractive force available is obtained by multiplying truck’s speed by the factor 0.0003714 (Fitch, 1994) in order to obtain the required motor power as horse power (HP) or by 0.000498 for kW.

The following expressions are used to evaluate the truck’s fuel efficiency and the amount of CO₂ emitted (Torres, 1998; Zadek, 2010). Using the proper units, fuel efficiency can be expressed in km/l and CO₂ emission in kg/100km travelled. Thus,

\[
F_e = \frac{V}{P_t \times SC \times \frac{1}{\rho}}
\]

\[
CO_2 = \frac{D}{F_e} \times (HV \times EF)
\]

### Mexican Case Example. Field vs Calculated Data

According to the Mexican Transport Institute (IMT), vehicles transporting the highest load volume in México are those shown in Table I. These heavy-duty diesel trucks (HDDT) are the two-axle truck, three-axle truck, five-axle articulated and double. Of these vehicles, the five-axle articulated and the double stand out, both for transporting most of cargo in the country and for being the most overloaded units. As an example, in 2002, CO₂ emissions in México were estimated in 643,183,000 tons (SEMARNAIT, 2007). According to the greenhouse gases national emission inventory of 2002 (Mar, 2005), the total CO₂ emissions from the transport sector were 111,942,170 tons. From this amount, 91% was emitted by trucks, 6% by airplanes, 2% by ships and the remaining 1% by railroad cars. This estimation was carried out (González, 2007) following the Good Practice Guidance and Uncertainty Management (GBPMI) of the Intergovernmental Panel on Climate Change (IPCC). Similar figures of CO₂ emissions in México were estimated in 2007 (Table I), these units carried 68.7% of total load moved on Mexican roads, representing 83.2% of total load economic value for that year. These trucks are also the most overloaded, with an average extra gross vehicle weight of 22.1% for the five-axle articulated and 31.1% for the double truck.

The truck overweight problem is associated with short term profits (EMFAC, 2007). But overloading is not necessarily associated with lower operating costs; on the long run an overloaded truck requires a more intensive maintenance. Additionally, an overloaded truck is more prone to be involved in accidents, causing more casualties (Chan, 2008) and road damage (Jacob and Foyer, 2010), and increases fuel consumption (Naguib, 2011). Due to this extra fuel consumption, truck overloading causes an increment on CO₂ emissions (Halenka and Uherek, 2010).

In 2002, CO₂ emissions in México were estimated in 643,183,000 tons (SEMARNAIT, 2007). According to the greenhouse gases national emission inventory of 2002 (Mar, 2005), the total CO₂ emissions from the transport sector were 111,942,170 tons. From this amount, 91% was emitted by trucks, 6% by airplanes, 2% by ships and the remaining 1% by railroad cars. This estimation was carried out (González, 2007) following the Good Practice Guidance and Uncertainty Management (GBPMI) of the Intergovernmental Panel on Climate Change (IPCC). Similar figures of CO₂ emissions in México were estimated in 2007 (Table I), these units carried 68.7% of total load moved on Mexican roads, representing 83.2% of total load economic value for that year. These trucks are also the most overloaded, with an average extra gross vehicle weight of 22.1% for the five-axle articulated and 31.1% for the double truck.

### Table I

#### VEHICLES TRANSPORTING THE HIGHEST LOAD VOLUME IN MÉXICO IN 2007

<table>
<thead>
<tr>
<th>Type of truck</th>
<th>Two axle (17.5)</th>
<th>Three axle (26)</th>
<th>Five axle articulated (45)</th>
<th>Double (66.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average gross vehicle weight</td>
<td>17.3</td>
<td>14.5</td>
<td>22.1</td>
<td>31.1</td>
</tr>
<tr>
<td>Percentage of total carried load on Mexican roads (2007)*</td>
<td>6.40</td>
<td>6.40</td>
<td>36.50</td>
<td>32.20</td>
</tr>
<tr>
<td>Percentage of total carried economic value on Mexican roads (2007)*</td>
<td>6</td>
<td>NA</td>
<td>54.70</td>
<td>28.5</td>
</tr>
</tbody>
</table>

* SCT (2009), Gutiérrez et al. (2008, 2010).  
GVW: gross vehicle weight (PROY-NOM-012-SCT-2-2003), NA: not available.

### Table II

#### EXPERIMENTAL DATA

<table>
<thead>
<tr>
<th>Route</th>
<th>Vehicle class</th>
<th>GVW (tons)</th>
<th>Load level</th>
<th>Travelled distance (km)</th>
<th>Average velocity (km/h)</th>
<th>Fuel efficiency (km/l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>double</td>
<td>30</td>
<td>empty</td>
<td>249</td>
<td>90</td>
<td>2.57</td>
</tr>
<tr>
<td>2</td>
<td>double</td>
<td>85</td>
<td>empty</td>
<td>646</td>
<td>70</td>
<td>1.40</td>
</tr>
<tr>
<td>3</td>
<td>double</td>
<td>30</td>
<td>empty</td>
<td>46</td>
<td>60</td>
<td>2.45</td>
</tr>
<tr>
<td>4</td>
<td>double</td>
<td>85</td>
<td>empty</td>
<td>452</td>
<td>60</td>
<td>1.40</td>
</tr>
<tr>
<td>5</td>
<td>double</td>
<td>30</td>
<td>empty</td>
<td>110</td>
<td>60</td>
<td>2.50</td>
</tr>
<tr>
<td>6</td>
<td>double</td>
<td>35</td>
<td>loaded</td>
<td>789</td>
<td>70</td>
<td>2.48</td>
</tr>
<tr>
<td>7</td>
<td>faa</td>
<td>20</td>
<td>empty</td>
<td>789</td>
<td>80</td>
<td>2.55</td>
</tr>
<tr>
<td>8</td>
<td>faa</td>
<td>25</td>
<td>loaded</td>
<td>885</td>
<td>80</td>
<td>2.30</td>
</tr>
<tr>
<td>9</td>
<td>faa</td>
<td>20</td>
<td>empty</td>
<td>885</td>
<td>90</td>
<td>2.60</td>
</tr>
</tbody>
</table>

GVW: gross vehicle weight (PROY-NOM-012-SCT-2-2003), faa: five axle articulated.
speed, road type and road surface conditions represent an impractical and extremely costly effort. Thus, fuel efficiency was obtained for nine test runs. At the end of each test, the amount of consumed fuel was divided by the traveled distance. Experimentally obtained fuel efficiency for the test runs, is shown in Table II. Table III shows the percentage of total traveled distance, indicating slope and surface road condition, for each route. Data taken from Tables II and III were used to calculate fuel efficiency and CO₂ emissions by applying Eqs. 7 to 12. Table III also shows these average fuel efficiencies.

Chebyshev’s intervals, calculated with different percentages of confidence level and values, are presented on Table IV. Data sources for these calculations are experimental trips (n₁), truck’s company (n₂) and those calculated (n₃) with Eqs. 7 to 12. The calculated Chebyshev’s intervals contain the sample means experimentally and truck’s company ones, which also can be seen in Table IV. In particular for k= 3.16 there is a likelihood that 90 out of 100 samples of size 9, contains the population mean. Variation coefficients are calculated using the expression

\[ CV = \frac{s}{\bar{x}} \]  

(13)  


For each set of data, the variation coefficients are CV₁, calculated from experimental data (n₁= 9); CV₂, calculated from truck’s company data (n₂= 52); and CV₃, calculated from theoretical data (n₃= 9)

Thus,

\[ n₁ \rightarrow CV₁ = \frac{0.4896}{2.25} = 0.2176 < 1 \]

\[ n₂ \rightarrow CV₂ = \frac{0.4911}{2.0378} = 0.2410 < 1 \]

\[ n₃ \rightarrow CV₃ = \frac{0.647}{1.926} = 0.3359 < 1 \]

The variability coefficients CV₁, CV₂ and CV₃ is <1. This value implies that there is low data variability from their respective mean values. Also CV₁>CV₂>CV₃, means that the calculated data are more heterogeneous than the experimental and truck’s company sample data.

For different levels of confidence, theoretical data Chebyshev’s intervals, contain the experimental and company samples averages. Mean values for the experimental data, company sample data, and that given by theoretical data, are 2.25, 2.03 and 1.92, respectively. Thus, formulations that yield the theoretical data are validated with a sample, n = 9, within a 90% confidence interval.

*Statistical Inference for calculating fuel efficiency (km/l)*

It has been established that Eqs. 7 to 12, validated in previous section, allow to simulate truck’s fuel consumption within a 90% confidence interval. So, using these expressions, the following two conditions were simulated: 1) keeping truck’s weight and speed constants, traveling on hilly terrain managing slopes of 1, 1.5, 2 and 2.5% (Avila, Arriaga, 2002), and 2) keeping the slope constant, trucks traveling at different loads (loaded and overloaded) and speeds (60, 80 and 110 km/h).

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### TABLE III

PERCENTAGE OF TOTAL TRAVELLED DISTANCE AT A CERTAIN SLOPE AND SURFACE ROAD CONDITION

<table>
<thead>
<tr>
<th>Type of terrain*</th>
<th>Flat +</th>
<th>Flat</th>
<th>Hilly</th>
<th>Hilly</th>
<th>Mountainous</th>
<th>Fuel efficiency (km/l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Route</td>
<td>Good (%)</td>
<td>Regular (%)</td>
<td>Good (%)</td>
<td>Regular (%)</td>
<td>Good (%)</td>
<td>T</td>
</tr>
<tr>
<td>1</td>
<td>90</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>2.17</td>
</tr>
<tr>
<td>2</td>
<td>81</td>
<td>17</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1.06</td>
</tr>
<tr>
<td>3</td>
<td>85</td>
<td>0</td>
<td>15</td>
<td>0</td>
<td>2</td>
<td>2.66</td>
</tr>
<tr>
<td>4</td>
<td>53</td>
<td>45</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0.70</td>
</tr>
<tr>
<td>5</td>
<td>73</td>
<td>27</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2.43</td>
</tr>
<tr>
<td>6</td>
<td>63</td>
<td>33</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>1.81</td>
</tr>
<tr>
<td>7</td>
<td>63</td>
<td>0</td>
<td>33</td>
<td>0</td>
<td>4</td>
<td>2.38</td>
</tr>
<tr>
<td>8</td>
<td>60</td>
<td>0</td>
<td>35</td>
<td>0</td>
<td>5</td>
<td>2.02</td>
</tr>
<tr>
<td>9</td>
<td>60</td>
<td>0</td>
<td>35</td>
<td>0</td>
<td>5</td>
<td>2.06</td>
</tr>
</tbody>
</table>


---

### TABLE IV

CHEBYSHEV’S INTERVALS

<table>
<thead>
<tr>
<th>Significance level</th>
<th>Confidence level (%)</th>
<th>k values</th>
<th>Theoretical</th>
<th>Experimental</th>
<th>Truck company data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>99</td>
<td>10</td>
<td>(0.40-5.0)</td>
<td>(0.65-3.85)</td>
<td>(1.35-2.70)</td>
</tr>
<tr>
<td>5</td>
<td>95</td>
<td>4.47</td>
<td>(0.99-2.87)</td>
<td>(1.53-2.96)</td>
<td>(1.73-3.22)</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
<td>3.16</td>
<td>(1.26-2.59)</td>
<td>(1.74-2.75)</td>
<td>(1.81-2.24)</td>
</tr>
<tr>
<td>15</td>
<td>85</td>
<td>2.58</td>
<td>(1.38-2.47)</td>
<td>(1.83-2.66)</td>
<td>(1.85-2.20)</td>
</tr>
</tbody>
</table>

---

### TABLE V

EXTRA CO₂ EMISSIONS (KG/100KM) FOR FIVE AXLE ARTICULATED AND DOUBLE TRUCKS, FOR DIFFERENT LOAD, SLOPE AND SPEED; TERRAIN: HILLY; ROAD SURFACE: REGULAR

<table>
<thead>
<tr>
<th>Slope (%)</th>
<th>Speed (km/h)</th>
<th>GVW (ton)</th>
<th>Fuel efficiency (%): 45 (65)</th>
<th>Extra CO₂ emitted due to overloading (%)</th>
<th>GVW (ton)</th>
<th>Extra CO₂ emitted due to overloading (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>60</td>
<td>237</td>
<td>315</td>
<td>39.66</td>
<td>66.5</td>
<td>98</td>
</tr>
<tr>
<td>0.85</td>
<td>80</td>
<td>270</td>
<td>369</td>
<td>36.60</td>
<td>338</td>
<td>486</td>
</tr>
<tr>
<td>1.0</td>
<td>110</td>
<td>330</td>
<td>437</td>
<td>32.42</td>
<td>376</td>
<td>633</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Worst case</td>
<td>61.85</td>
<td>445</td>
<td>614</td>
</tr>
<tr>
<td>1.5</td>
<td>60</td>
<td>283</td>
<td>397</td>
<td>40.28</td>
<td>406</td>
<td>585</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Worst case</td>
<td>61.85</td>
<td>444</td>
<td>632</td>
</tr>
<tr>
<td>2.0</td>
<td>80</td>
<td>316</td>
<td>435</td>
<td>37.65</td>
<td>412</td>
<td>713</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Worst case</td>
<td>63.29</td>
<td>444</td>
<td>632</td>
</tr>
<tr>
<td>2.5</td>
<td>110</td>
<td>422</td>
<td>569</td>
<td>34.83</td>
<td>503</td>
<td>713</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Worst case</td>
<td>60.58</td>
<td>503</td>
<td>713</td>
</tr>
<tr>
<td>3.0</td>
<td>60</td>
<td>329</td>
<td>463</td>
<td>40.72</td>
<td>473</td>
<td>685</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Worst case</td>
<td>65.08</td>
<td>473</td>
<td>685</td>
</tr>
<tr>
<td>3.5</td>
<td>80</td>
<td>407</td>
<td>567</td>
<td>39.31</td>
<td>541</td>
<td>785</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Worst case</td>
<td>57.8</td>
<td>579</td>
<td>831</td>
</tr>
<tr>
<td>4.0</td>
<td>110</td>
<td>467</td>
<td>635</td>
<td>35.97</td>
<td>648</td>
<td>913</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Worst case</td>
<td>57.68</td>
<td>648</td>
<td>913</td>
</tr>
</tbody>
</table>
**Estimation of CO$_2$ emissions (kg/100km)**

CO$_2$ emissions were calculated according to Ecs 7-12. Table V presents CO$_2$ levels (kg/100km) emitted by five axle articulated and double trucks traveling on a regular condition road surface, negotiating various slopes at different speeds and load levels. On this table it can also be seen the emission's level increase when a truck is traveling overloaded or speeding. From this information it can be evaluated one of the worst cases for CO$_2$ emissions, that is, when a truck travels overloaded and speeding. In this situation a truck generates in the range of 60% more CO$_2$ than allowed by the operating standards set by the authority.

**Results**

The methodology presented allows simulating different scenarios. These scenarios reflect with a very good approximation the extra pollution that speeding and overloaded trucks emit. So, based on the available data related with the amount of overloaded and speeding trucks, as well as the trend of growth in the number of units, estimation of CO$_2$ emitted by trucks can be carried out. For the Mexican case, Figure 1 shows the annual CO$_2$ (kg/100km) emitted by five axle and double trucks, both with weight and average speed regulated and not regulated. Figure 2 shows the results of annual CO$_2$ (kg/100km) emitted by both types of trucks, both with weight and average speed regulated and not regulated.

**Conclusions**

1. A methodology for estimating CO$_2$ emissions from overloaded and speeding heavy duty vehicles, based on Chebyshev’s Theorem was presented.
2. The methodology presented ensures, within a certain statistical confidence level, validation of fuel efficiency data coming from different sources.
3. For the example presented, the theoretical calculated Chebyshev’s intervals, contain experimental and sampled (truck’s company) data means.
4. Using the validated model, estimation within a 90% confidence level of CO$_2$ emissions produced by overloaded and speeding trucks were carried out.
5. Considering the growth in vehicle numbers and the overloading and speeding in the past nine years in México, it was estimated that due to such overloading and speeding, an average of 60% more CO$_2$ was annually emitted.

**REFERENCES**


APLICACIÓN DEL TEOREMA DE CHEBYSHEV PARA ESTIMAR LAS EMISIONES DE CO₂ POR SOBRECARGA DE LAS UNIDADES DE TRANSPORTE TERRESTRE DE CARGA

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RESUMEN

El número de camiones cargados que sobrepasan los límites autorizados en México es alto. La sobrecarga propicia emisiones de CO₂ adicionales, aumentando significativamente la cantidad de contaminantes emitidos a la atmósfera. Medir estas emisiones se complica debido a que resulta obligado conocer en todo momento las condiciones de operación de los camiones, tales como velocidad, aceleración, carga, rugosidad de la carretera y pendiente. Registrar estos datos para todos los vehículos en todo momento es poco práctico. En este trabajo se presenta una metodología para estimar las emisiones de los camiones que circulan sobrecargados y/o a exceso de velocidad, basada en la aplicación del teorema de Chebyshev. El uso de modelos teóricos, permite estimar dentro de un cierto nivel de significancia estadística el consumo de combustible y las emisiones de CO₂. Utilizando la metodología propuesta se valida un modelo de camión con datos experimentales mínimos necesarios. Una vez que este modelo es validado, es posible simular varias condiciones de operación, incluyendo la sobrecarga del camión y la velocidad. Para esto, se utilizan modelos simplificados del consumo de diesel en los vehículos pesados. De los resultados obtenidos, se concluye que con el método propuesto, basado en el teorema de Chebyshev, se pudo estimar que, dentro de un límite de confiabilidad del 90%, debido a sobrecarga y velocidad excesiva, los vehículos pesados en México, emiten alrededor de 60% más contaminantes, de lo que emitirían si se cumplieran dentro de los límites establecidos en los reglamentos.

APLICACIÓN DO TEOREMA DE CHEBYSHEV PARA ESTIMAR AS EMISSIONS DE CO₂ POR SOBRECARGA DAS UNIDADES DE TRANSPORTE TERRESTRE DE CARGA

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RESUMO

O número de caminhões carregados que sobrepasam os limites autorizados no México é alto. A sobrecarga propicia emissões de CO₂ adicionais, aumentando significativamente a quantidade de contaminantes emitidos a atmosfera. Medir essas emisiones se complica devido a que resulta obrigado conhecer as condições de operação dos caminhões em todo momento, como a velocidade, a aceleração, a carga, a rugosidade da estrada e a pendente. Registrar estes dados para todos os veículos em todo momento é pouco prático. Neste trabalho se apresenta uma metodologia para estimar as emissões dos caminhões que circulam sobrecarregados, baseada na aplicação do Teorema Chebyshev. O uso de modelos teóricos, permite estimar dentro de um certo nível de significância estatística o consumo de combustível e as emissões de CO₂. Utilizando a metodologia proposta se valida um modelo de caminhão com dados experimentais mínimos necessários. Uma vez que este modelo seja valido, é possível simular várias condições de operação, incluindo a sobrecarga do caminhão e a velocidade. Para isto, se utilizam modelos simplificados do consumo de diesel nos veículos pesados. Dos resultados obtidos, se conclui que com o método proposto, baseado no teorema de Chebyshev, se pode estimar que, dentro de um limite de confiabilidade do 90%,vido a sobrecarga e velocidade excessiva, os veículos pesados no México, emitem ao redor de 60% mais contaminantes, do que emitiriam se se cumprissem dentro de os limites estabelecidos nos regulamentos.

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