GROWTH MODELS OF PEACH FRUIT Prunus persica (L) IN THREE

HANDLING SYSTEMS

Joel Díaz Martínez, Isaias Chairez Hernández, J. Natividad Gurrola Reyes, José Bernardo Proal Nájera, Martha Celina González Güereca and Edmundo Castellanos Pérez

SUMMARY

This study was conducted in orchards located in Durango, México, during 2013, at a site with an average annual precipitation of 450mm and mean temperature of 17.7°C. The goal was to evaluate three non-linear models: logistic, monomolecular and Gompertz, to simulate peach fruit growth as measured by polar (PD) and equatorial (ED) diameters using the Creole peach. The first treatment (T1) received pruning, irrigation, fertilization (PIF) and application of organic insecticides. The second treatment (T2) received only PIF, and the control (T3) received only irrigation. Parameters measured included PD, ED and inflection points (IP). The models were evaluated using the square sum of errors (SSE), coefficient of determination (R^2) , and the Akaike criterion; the Student test t was used to compare the coefficients among models. The smallest values of SSE and Akaike indicated that the monomolecular model was the best fit for PD and ED with R^2 = 0.9998, 0.9997 and 0.9998 for T1, T2 and T3, respectively. The largest diameter was found in model for T1 and differed from the models for T2 and T3. Simulation of peach fruit growth with the monomolecular model allows for the description of diameter growth rates with reference to IPs and should facilitate the planning of agronomic and pest control tasks

Introduction

Mexican peach cultivars cover an area of 43942ha with a production rate of 4ton ha⁻¹. The yield per ha varies according to region, variety, agronomic handling techniques, climate, insects and diseases. Within Mexico, the states of Zacatecas, Michoacán, México, Puebla, Chihuahua, Morelos and Durango are the primary peach producers. Mexico's per capita peach intake is 1.5kg, it is the twelfth largest peach producer in the world, after Chile, Argentina and Brazil in Latin America, and production has increased in recent years (SAGARPA, 2013). However, problems with handling, nutrition, phytosanitation and commercialization have hindered increases in the quantity and quality of peaches. Knowledge of peach phenology and agroclimatic variables will confer the potential for local adaptation of peach crops.

Zucconi (1986) established the existence of three stages of peach fruit growth and development. The first stage encompasses full flower development through endocarp hardening, in which mitosis takes place during the first three weeks and then rapidly declines. The second stage is characterized by slow growth of the mesocarp, cessation of general elongation and lignification of the endocarp. During the third stage the fruit grows rapidly; cellular elongation continues and the intercellular spaces almost disappear (Marini and Reighard, 2008). Peach fruit phenology, its mathematical growth models and agroclimatic variables are important tools because they describe the rate of growth during sprouting, flowering and fruiting. This information is vital for handling peach orchards and for the incorporation of new varieties. Several studies have focused on peach fruit phenology. Interestingly, Medina-Torres (2000) found a mean of chill units and a thermal time accumulation (TTA) of 226.8 and 1207 of the peach variety CP-9216 from January 20th to May 20^{th} , with a period of 120 days. Lott (1942) characterized the phenology of the 'Hale Haven' peach in three stages in addition to describing the effect of applying nitrogen soda to peach trees. Similarly, a full description of this peach variety from germ sprout through flowering and fruit development was provided by Donoso et al. (2008) for certain regions of Chile; however, they did not develop mathematical models. Casierra-Posada et al. (2004) studied peach phenology, measured wet and dry weights and calculated PD/ED ratios over time using third degree

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RESUMEN

Este estudio fue realizado en huertas localizadas en Durango, México, durante el 2013, en un sitio con precipitación promedio anual de 450mm y temperatura media de 17,7°C. El objetivo fue seleccionar tres modelos no lineales: logístico, monomolecular y Gompertz, para simular el crecimiento del fruto de durazno en base al diámetro polar (PD) y ecuatorial (ED) usando el durazno criollo. El primer tratamiento (TI) recibió poda, irrigación, fertilización (PIF) y aplicación de insecticidas orgánicos. El segundo tratamiento (T2) recibió solo PIF y el control (T3) recibió solo irrigación. Los parámetros que se midieron fueron PD, ED y puntos de inflexión (IP). Los modelos fueron evaluados usando la suma de errores al cuadrado (SSE), coeficiente de determinación (R^2), y el criterio de Akaike; la t de Student fue usada para comparar los coeficientes entre los modelos. Los valores más pequeños de SSE y Akaike indicaron que el modelo monomolecular fue el que mejor se ajustó en PD y ED con $R^2=0,9998$; 0,9997 y 0,9998 para T1, T2 y T3 respectivamente. El diámetro más grande fue encontrado en T1 y fue diferente a los modelos para T2 y T3. La simulación del crecimiento del fruto de durazno con el modelo monomolecular permite describir las tazas de crecimiento del diámetro en referencia a los IP y puede facilitar la planeación de labores agronómicas y de control de plagas.

MODELOS DO CRESCIMENTO DO FRUTO DE PÉSSEGO Prunus pérsica (L) EM TRÊS SISTEMAS DE MANEJO

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RESUMO

Este estudo foi realizado em hortas localizadas em Durango, México, durante 2013, em um local com precipitação média anual de 450mm e temperatura média de 17,7°C. O objetivo foi selecionar três modelos não lineares: logístico, monomolecular e Gompertz, para simular o crescimento do fruto de pêssego com base no diâmetro polar (PD) e equatorial (ED) usando o pêssego crioulo. O primeiro tratamento (T1) recebeu poda, irrigação, fertilização (PIF) e aplicação de inseticidas orgânicos. O segundo tratamento (T2) recebeu somente PIF e o controle, (T3) recebeu somente irrigação. Os parâmetros que se mediram foram PD, ED e pontos de inflexão (IP). Os modelos foram avaliados

polynomials and found correlations >0.97. Gutiérrez-Acosta et al. (2008) carried out a characterization of the 'Ana' peach cultivar quantifying fruit number, PD, ED and fruit stone and pulp thickness from 2000 to 2004 using descriptive statistics, correlation coefficients and a simple linear regression among variables. However, although Gutiérrez-Acosta et al. (2008) and Casierra-Posada et al. (2004) used mathematical models, they did not recognize the three stages described by Zucconi (1986).

The morphological characterization and simulation of different peach fruits has also been reported by Álvarez and Boche (1999), who measured the perpendicular equatorial diameter of late-season nectarines (c.v. Sun Grand) and modeled the diameters with monomolecular, logistic and Gompertz models (Paine et al., 2012). They found that the logistic model had the lowest mean square error (MSE) with a correlation coefficient between 0.993 and 0.997. In contrast, in Mexico State, Rojas-Lara et al. (2008) applied four non-linear regressions including the double sigmoid logistic, exponential, logistic, Michaelis-Menten and monomolecular models to estimate 'manzano' hot pepper (Capsicum pubescens R & P) growth under greenhouse conditions during two sampling periods in 2004 and 2005, using fresh fruit weight as the dependent variable and fruit growth time as the independent variable. There were significant differences

usando a soma de erros ao quadrado (SSE), coeficiente de determinação (R^2), e o critério de Akaike; a t de Student foi usada para comparar os coeficientes entre os modelos. Os valores menores de SSE e Akaike indicaram que o modelo monomolecular foi o que melhor se ajustou em PD e ED com R^2 =0,9998; 0,9997 e 0,9998 para T1, T2 e T3 respectivamente. O diâmetro maior foi encontrado em T1 e foi diferente aos modelos para T2 e T3. A simulação do crescimento do fruto de pêssego com o modelo monomolecular permite descrever as taxas de crescimento do diâmetro em referência aos IP e pode facilitar o planejamento de labores agronômicas e de controle de pragas.

between the two periods, and the monomolecular model provided the best estimate of fresh fruit weight for both periods. Pear fruit growth was evaluated by Arenas Bautista et al. (2012) under two drip irrigation systems; there were no significant differences between treatments and the logistic regression provided the best fit for fruit growth. Pear (Pyrus communis L) growth and physical and physiological characterization without mathematical modeling was conducted by Parra-Coronado et al. (1998). In tomatoes, a growth analysis of three hybrid fruits (Solanum L and S. copersicum L.) under greenhouse conditions was conducted by Ardila et al. (2011) using logistic models; the independent variable was the TTA with a base tempera-

ture of 10°C. A description of the full growth of sweet orange fruit (*Citrus sinensis*, Valencia variety) was reported by Avanza *et al.* (2004) using logistic, Gompertz and monomolecular models. The authors concluded that the monomolecular model was the most suitable one.

Non-linear models have been used to describe the phenology of nectarines, hot peppers, tomatoes, pears and oranges. Thus, double sigmoid models such as the logistic, monomolecular and Gompertz equations are useful and adequate tools for simulating peach fruit phenology. The objective of this study was to compare these three different double sigmoid models: logistic, monomolecular and Gompertz, to describe the phenology of peach fruit growth as a function of growth rate and growth diameters in three peach tree handling systems.

Materials and Methods

The experimental data used for model simulations was obtained from three mixed Creole peach orchards with three different handling systems in the community of San Nicolás de Arriba, Santiago Papasquiaro, Durango, México. This community is located at 25°02' 38"N and 105°25'09"W, at an elevation of 1713masl (Figure 1). It has a dry temperate sub-humid seasonal climate with an average precipitation of 450mm distributed primarily during the summer, with only 5-10% of precipitation occurring during the winter and mean temperature of 17.7°C (García, 1990).

The harvest year was 2013. In orchard T1, the agronomic handling techniques included a winter pruning system with the removal of dead, diseased or broken branches in February. followed by an application of Bordeaux mixture at a concentration of 250g of copper (II) sulfate (CuSO₄) and 2kg of slaked lime (Ca(OH)₂) diluted in 51 of water; the mixture was used as a fungicide and applied to trees at a height of 1m above ground. A bowl for auxiliary irrigation was constructed in March. Fertilizer was applied in April. Composted manure of medium composition from the same region was applied to the trees at a rate of 7kg·m² in addition to a chemical fertilizer with a 25-25-25 NPK ratio divided into three bimonthly

doses. The first dose was in May; beginning in June it was combined with a foliar fertilizer at a dilution of 250ml of fertilizer in 151 water. The summer pruning was conducted in June to eliminate vigorous growth that causes shading.

Peach cultivars were irrigated to avoid a moisture deficit from March until the rainy season in July at a rate of 1001 water every 10 or 12 days per tree. Alternative methods were used to control insects. Delta traps with species-specific lures (male pheromones) were set out in June to prevent reproduction and fertilization of female insects; these traps had a longevity of 35 days. In addition, insect control methods included biodegradable or environmentally friendly products such as extracts of vegetable oils (e.g., neem oil) and pesticides of botanical origin (garlic and oregano); these products were applied every 15 days from the beginning of June to the end of September. For aerial control of disease, 1kg of Captan® per ha was applied every 15 days from June to September; this prevented disease during the rainy season. Orchard T2 received the same general treatment as orchard T1, but there was no protection against insects and no aerial disease control. Orchard T3 was the control. Although crop management, pruning, chemical fertilization, and insect and disease control were not applied in T3, the planned irrigation was regularly performed. Five trees were selected from each treatment and from each tree, five fruit were randomly chosen.



Figure 1. Santiago Papasquiaro, Durango, Mexico. The circle indicates the orchard location.

Measurements began 21 days after full bloom (DAFB), which occurred on April 6th 2013, and continued until fruit were ready for harvesting. Fruit growth, measured as polar and perpendicular equatorial diameters, was recorded. The measurement period lasted for 18 weeks. The final measurement occurred on August 24th 2013, when the fruit reached the required ripening point for harvesting. TTA with a base temperature of 10°C from full bloom until fruit were ready to harvesting, precipitation from January to August and chill units from January to February were obtained from the climatic station Las Margaritas, Santiago Papasquiaro of INIFAP.

A repeated measures analysis of variance (ANOVA) with a 3×5 factorial design was conducted. The first factor was the treatment (handling systems); the second factor was the trees. The Gauss-Markov postulates for this test were met. The least significant difference (LSD) was used for pairwise comparisons. The growth kinetics of fruit diameters were obtained for the logistic (1), monomolecular (2), and Gompertz (3) models. In each case, f(t) represents the fruit diameter measurement in mm and t the time in DAFB. The constant a is related to the final diameter and the constants b, c, d and e are associated with growth rates. For this analysis, the Gauss-Newton algorithm with the Marquardt

correction was used. The iterative computing method demands the introduction of initial values for the coefficients; thus, it was necessary to provide approximate estimates of the coefficients. To measure the goodness of fit and compare the models, it was necessary to calculate the SSE, the Akaike criterion and the coefficient of determination (R²). Mean model coefficients were compared using the Student's t test for each coefficient of each model. The first derivative of each model was calculated to determine the growth rate and the inflection point (IP); the diameter growth rates of PD/ED were calculated for each individual model. The software used was STATISTICA 7 (StatSoft, Inc. 2004, Tulsa, OK, USA) and MATLAB 7.0 (The MathWorks, Inc. Natick, MA, USA).

$$f(t) = \frac{a}{1 + e^{-(b + ct + dt^2 + et^3)}}$$
(1)

$$f(t) = a\left(1 - e^{-\left(b + ct + dt^2 + et^2\right)}\right)$$
(2)

$$f(t) = a * e^{\left(-e^{\left(b+ct+dt^{2}+ct^{3}\right)}\right)}$$
(3)

Results and Discussion

Table I, shows the polar and equatorial diameters of the peach fruit for the three treatments. Measurements were taken every two weeks for 140 days beginning on April 27th

TABLE I POLAR AND EQUATORIAL DIAMETER MEASUREMENTS IN GROWING PEACH FRUIT

DAEB	Р	olar diamete	er	Equatorial diameter						
υάγο	T1 (mm)	T2 (mm)	T3 (mm)	T1 (mm)	T2 (mm)	T3 (mm)				
21	17.48 a	16.92 a	16.96 a	12.96 b	13.52 b	11.96 a				
35	31.12 c	31.00 c	27.72 b	27.04 e	27.04 e	24.00 c				
55	33.20 d	33.92 e	31.08 c	28.04 f	27.48 ef	26.04 d				
70	35.04 e	35.56 f	33.64 d	29.04 g	29.00 h	27.04 e				
84	36.76 f	36.72 g	35.48 e	29.54 g	30.96 i	28.04 f				
98	40.48 h	40.16 h	37.80 g	34.52 jk	37.48 k	32.00 i				
112	47.20 jk	46.64 k	42.76 i	42.48 n	40.96 m	38.00 k				
126	51.76 m	48.4 4m	46.56 j	47.00 p	43.48 o	40.04 1				
140	54.92 n	51.64 np	47.8 kľ	52.48 a	46.48 p	43.00 mn				

T1: agronomic and pest management, T2: agronomic management, T3: without agronomic or pest management, DAFB: days after full flowering, and LSD: least significant difference. Different small letters in superscript indicate significant differences p<0.05 with LSD test. 2013, 21 days after full blossom, and ending on August 24th when the peach fruit achieved their maximum size and were ready to be harvested. The proposed model for the experimental design met the Gauss-Markov assumptions of normality, homogeneity of variance and independence. The ANOVA showed significant differences among treatments and a significant interaction between treatment and time for both diameters. Significant differences were observed among the different time periods and treatments, with the exception of the equatorial diameter of T1, which corresponded to the second growth stage and was characterized by slow growth. Significant differences were exhibited among the three treatments and the two diameters at day 140. This result indicated that both the agronomical and pest management treatments were effective.

Table II shows the model coefficients for the polar and equatorial diameters of peach fruit after flowering for T1, T2 and T3. Based on the SSE, the Akaike criterion and the R² values, the best model for the polar and equatorial diameters of T1, T2 and T3 was the mono-molecular model. Therefore, the chosen model to calculate the growth rate and the equatorial and polar diameter ratio was the monomolecular model (Figure 2a).

Table III compares the coefficients of the growth models $(t=2.11, df=16 \text{ and } p \le 0.05).$ Values of t >2.11 show significant differences between coefficients. Values of t in a coefficient between T1 and T2 as well as between T1 and T3 showed differences in all handling systems in the three models. However, there were no significant differences between T2 and T3. No significant differences for coefficients b, c, d or e were found. Based on the model comparisons, T1 was significantly different from both T2 and T3. Polar and equatorial diameters were larger in the T1 treatment due to the agronomical practices and protection from pests (Table II).

Figure 2b shows the IP in the minimum and maximum of first derivative from the monomolecular model obtained in MATLAB 7.0. These points show the change in the peach fruit growth rate for the PD (continuous lines) and ED (dotted lines). The PD values revealed sustained growth until day 63 for T1, T2 and T3. Subsequently, there was a period of lower growth on average, from day 63 until day 107, 104 and 110 for T1, T2 and T3, respectively, with a mean of 107. Later, a rapid growth phase continued until day 140 when T1 reached its maximum diameter followed by T2 and T3. Equatorial diameter maintained sustained growth until day 66, 60 and 64 for T1, T2 and T3, respectively, with a mean of 63. Subsequently, a period of slow growth continued until day 109, 107 and 105 for T1, T2 and T3, respectively. Rapid growth then continued until day 140 when T1 reached its maximum diameter followed by T2 and T3. It is notable that in both diameter

TABLE	Π

TADLE II													
MODEL	COEFFICIENTS	FOR	PEACH	FRUIT	DIAMETER	BY	TREATMENT						

а		Polar diameter	SSE	\mathbb{R}^2	Akaike
	T1	$\frac{54.33}{1+e^{-(-3.2861+0.172t-0.00256t^2+0.000013t^3)}}$	6.336	0.99901	7.841
Logístic	T2	$\frac{50.39}{1+e^{-(-3.8959+0.2166t-0.00328t^2+0.000017t^3)}}$	6.949	0.99886	8.673
	Т3	$\frac{47.93}{1+e^{-(-2.9571+0.1557t-0.00223t^2+0.000011t^3)}}$	2.585	0.99952	-0.226
	T1	$55.30 \left(1 - e^{-(-0.9057 + 0.0867t - 0.00147t^{2} + 0.000007164t^{3})} \right)$	2.095	0.99967	-2.117
Monomolecular	T2	$51.18 \left(1 - e^{-\left(-1.2390 + 0.1104 - 0.00172t^2 + 0.0000905t^3 \right)} \right)$	3.681	0.99939	2.955
	Т3	$48.61 \left(1 - e^{-(-0.7695 + 0.0792t - 0.00118t^{2} + 0.00006256t^{3})} \right)$	1.610	0.99970	-4.488
	T1	$54.74e^{\left(-e^{\left(2.0052-0.1262(+0.00192t^{2}+0.0000986t^{3})\right)}\right)}$	3.896	0.99939	3.466
Gompertz	Т2	$50.73e^{\left(-e^{\left(2.4536-0.15864+0.00247t^2-0.00001t^3\right)}\right)}$	5.058	0.99917	5.815
	Т3	$48.20e^{\left(-e^{\left(1.7787-0.0145(+0.00167t^2-0.00000854t^3)\right)}\right)}$	1.925	0.99964	-2.877
b		Polar diameter	SSE	R ²	Akaike
		50.92	16 725	0.00668	16.577
	T1	$1 + e^{-(-3.9047 + 0.11974t - 0.00303t^2 + 0.000015t^3)}$	16./25	0.99008	
Logistic	T1 T2	$\frac{1 + e^{-(-3.9047 + 0.11974t - 0.00303t^{2} + 0.000015t^{3})}}{44.39}$ $\frac{44.39}{1 + e^{-(-4.4340 + 0.2524t - 0.00414t^{2} + 0.000022t^{3})}}$	14.832	0.99685	15.496
Logistic	T1 T2 T3	$\frac{1 + e^{-(-3.9047 + 0.11974t - 0.00303t^{2} + 0.000015t^{3})}}{44.39}$ $\frac{44.39}{1 + e^{-(-4.4340 + 0.2524t - 0.00414t^{2} + 0.000022t^{3})}}{41.72}$ $\frac{41.72}{1 + e^{-(-4.2639 + 0.2296t - 0.00356t^{2} + 0.000018t^{3})}}$	16.725 14.832 6.349	0.99685 0.99838	15.496 7.859
Logistic	T1 T2 T3 T1	$\frac{1+e^{-(-3.9047+0.11974t-0.00303t^{2}+0.000015t^{3})}}{44.39}$ $\frac{44.39}{1+e^{-(-4.4340+0.2524t-0.00414t^{2}+0.00002t^{3})}}{\frac{41.72}{1+e^{-(-4.2639+0.2296t-0.00356t^{2}+0.000018t^{3})}}$ $52.46\left(1-e^{-(-1.0459+0.0912t-0.00147t^{2}+0.00000772t^{3})}\right)$	16.725 14.832 6.349 6.460	0.99685 0.99838 0.99871	15.496 7.859 8.015
Logistic	T1 T2 T3 T1 T2	$\frac{1+e^{-(-3.9047+0.11974t-0.00303t^{2}+0.000015t^{3})}}{1+e^{-(-4.4340t+0.2524t-0.00414t^{2}+0.000022t^{3})}}$ $\frac{41.72}{1+e^{-(-4.2639+0.2296t-0.00356t^{2}+0.000018t^{3})}}$ $52.46\left(1-e^{-(-1.0459+0.0912t-0.00147t^{2}+0.00000772t^{3})}\right)$ $45.09\left(1-e^{-(-1.4343+0.1248t-0.00211t^{2}+0.000012t^{3})}\right)$	16.725 14.832 6.349 6.460 8.014	0.99685 0.99838 0.99871 0.99830	15.496 7.859 8.015 9.956
Logistic	T1 T2 T3 T1 T2 T2 T3	$\frac{1+e^{-(-3.9047+0.11974t-0.00303t^{2}+0.000015t^{3})}}{1+e^{-(-4.4340+0.2524t-0.00414t^{2}+0.00002t^{3})}}$ $\frac{41.72}{1+e^{-(-4.2639+0.2296t-0.00356t^{2}+0.000018t^{3})}}$ $52.46\left(1-e^{-(-1.0459+0.0912t-0.00147t^{2}+0.00000772t^{3})}\right)$ $45.09\left(1-e^{-(-1.4343+0.1248t-0.00211t^{2}+0.000012t^{3})}\right)$ $42.49\left(1-e^{-(-1.2812+0.1100t-0.00177t^{2}+0.00000949t^{3})}\right)$	16.725 14.832 6.349 6.460 8.014 2.770	0.99685 0.99838 0.99871 0.99830 0.99929	15.496 7.859 8.015 9.956 0.395
Logistic Monomolecular	T1 T2 T3 T1 T2 T3 T1	$\frac{1+e^{-(-3.9047+0.11974t-0.00303t^{2}+0.000015t^{3})}}{1+e^{-(-4.4340+0.2524t-0.00414t^{2}+0.00002t^{3})}}$ $\frac{41.72}{1+e^{-(-4.2639+0.2296t-0.00356t^{2}+0.000018t^{3})}}$ $52.46\left(1-e^{-(-1.0459+0.0912t-0.00147t^{2}+0.00000772t^{3})}\right)$ $45.09\left(1-e^{-(-1.4343+0.1248t-0.00211t^{2}+0.000012t^{3})}\right)$ $42.49\left(1-e^{-(-1.2812+0.1100t-0.00177t^{2}+0.00000949t^{3})}\right)$ $51.52e^{(-e^{(2.3575-0.1400t+0.00219t^{2}-0.0001t^{3})}}$	16.725 14.832 6.349 6.460 8.014 2.770 11.178	0.99685 0.99838 0.99838 0.99871 0.99830 0.99929 0.99778	15.496 7.859 8.015 9.956 0.395 12.950
Logistic Monomolecular Gompertz	T1 T2 T3 T1 T2 T3 T1 T2 T3 T1 T2	$\frac{1+e^{-(-3.9047+0.11974t-0.00303t^{2}+0.000015t^{3})}}{1+e^{-(-4.4340t+0.2524t-0.00414t^{2}+0.000022t^{3})}}$ $\frac{41.72}{1+e^{-(-4.2639+0.2296t-0.00356t^{2}+0.000018t^{3})}}$ $52.46\left(1-e^{-(-1.0459+0.0912t-0.00147t^{2}+0.00000772t^{3})}\right)$ $45.09\left(1-e^{-(-1.4343+0.1248t-0.00211t^{2}+0.000012t^{3})}\right)$ $42.49\left(1-e^{-(-1.2812+0.1100t-0.00177t^{2}+0.0000949t^{3})}\right)$ $51.52e^{(-e^{(2.3575-0.1400t+0.00219t^{2}-0.0001t^{3})})}$ $44.70e^{(-e^{(2.7893-0.1817t+0.00302t^{2}-0.0002t^{3})})}$	16.725 14.832 6.349 6.460 8.014 2.770 11.178 11.313	0.99685 0.99838 0.99838 0.99871 0.99830 0.99929 0.99778 0.99760	15.496 7.859 8.015 9.956 0.395 12.950 13.059
Logistic Monomolecular Gompertz	T1 T2 T3 T1 T2 T3 T1 T2 T3 T1 T2 T3	$\frac{44.39}{1+e^{-(-4.39047+0.11974t-0.00303t^{2}+0.000015t^{3})}} = \frac{44.39}{1+e^{-(-4.4340+0.2524t-0.00414t^{2}+0.000022t^{3})}} = \frac{41.72}{1+e^{-(-4.2639+0.2296t-0.00356t^{2}+0.00018t^{3})}} = 52.46\left(1-e^{-(-1.0459+0.0912t-0.00147t^{2}+0.00000772t^{3})}\right) = 45.09\left(1-e^{-(-1.4343+0.1248t-0.00211t^{2}+0.000012t^{3})}\right) = 42.49\left(1-e^{-(-1.2812+0.1100t-0.00177t^{2}+0.00000949t^{3})}\right) = 51.52e^{(-e^{(2.3575-0.1400t+0.00219t^{2}-0.0001t^{3})})} = 44.70e^{(-e^{(2.6359-0.1611t+0.00259t^{2}-0.0000t^{3})})} = 42.04e^{(-e^{(2.6359-0.1611t+0.00259t^{2}-0.0000t^{3})})} = 51.52e^{(-e^{(2.6359-0.1611t+0.00259t^{2}-0.0000t^{3})})} = 51.52e^{(-e^{(2.6359-0.1611t+0.00259t^$	16.725 14.832 6.349 6.460 8.014 2.770 11.178 11.313 4.301	0.99688 0.99685 0.99838 0.99871 0.99830 0.99929 0.99778 0.99760 0.99890	15.496 7.859 8.015 9.956 0.395 12.950 13.059 4.356

T1: agronomic and pest management, T2: agronomic management, T3: without agronomical or pest management, SSE: sum of squared errors, R^2 : coefficient of determination, and Akaike: N(ln(SCE/N))+(2k+1), where k is the number of parameters and N is the number of measurements.



Figure 2. Days after full bloom \times axis *versus* a: fruit diameter in mm (left axis), thermal time accumulation over 10°C (TTA, right axis), columns are chilly hours; b: growth rate; and c: polar/equatorial peach diameter in the monomolecular model.

types (PD and ED), T1 exhibited the largest diameter followed by T2 and T3. TTA from 6th April to 24thAugust were 1474, chill units 233 and precipitation 250mm (Figure 2a). The double sigmoid models and agroclimatic variables are adequate to simulate peach fruit phenology as measured by growth rate and growth diameter. Agroclimatic variables TTA 1474 and chill units 233 coincide with the study of Medina-Torres (2000) with 1207 and 226.8.

With respect to the final polar and equatorial diameters, Gutiérrez-Acosta et al. (2008) reported a PD of 48.9 to 65.1mm; the average of the actual data in this study was 54.92 and 51.6mm for T1 and T2, respectively. These values are within the range reported by Gutiérrez-Acosta et al. (2008); however, T3 had an average PD of 47.8mm, which is below this range. In the case of ED, only the average value of ED for T1 falls within the range of 52.23 to 69.1mm reported by Gutiérrez-Acosta et al. (2008). However, Lott (1942) reported higher PD and ED values of 60.5 and 62.8mm, respectively, compared with those found in this study.

The similarity between the results of this study and the results of Gutiérrez-Acosta et al. (2008) is likely because the peach is of the same variety; the difference compared with Lott (1942) is likely because the peach is a different variety, and nitrate of soda was applied to the trees. The differences among treatments in this study are related to pruning (T3), pruning, irrigation, fertilization (T2) and pruning, irrigation, fertilization and application of pest treatments (T1) (Table I).

Zucconi (1986) settled the presence of three growth stages in the peach fruit. The first is distinguished by an increment in fruit mitosis. Then, a slower growth than the first, and the third stage is characterized by an accelerated growth of the mesocarp (Marini and Reighard, 2008). A third degree polynomial simulation with an R²>0.98 was reported by Casierra-Posada et al. (2004), which yielded a weight of 0% at day 60 of DAFB (Figure 2 in Casierra-Posada et al. (2004)) and sustained growth until day 112. Subsequently, growth slowed until day 146. Between days 146 and 194, rapid growth occurred. As a consequence of the third degree polynomial model, growth continued; the first stage was 112-60=52 days, the second, 146-112=34 days, and the third of 194-146=48 days, for a total period of 134 days. The flowering period was reported by Gutiérrez-Acosta et al. (2008) as ranging from January 28th (Julian day 28) to February 8^{th} (39), and the harvest season was reported as ranging from August 9th (221) to August 16th (220). However, Lott (1942) determined an average growth period for the 'Hale Haven' peach of 56 days with the first stage beginning on April 14th, a second stage of 28 days and a third stage of 39 days with full maturation on July 7th. Moreover, Donoso et al. (2008) described the equality of the first and third stages. In this study, the first, second, third and total growth periods were 63, 44, 33 and 140 days, respectively

TABLE III COMPARISON AMONG COEFFICIENTS FROM GROWTH MODELS FOR PEACH FRUIT DIAMETER (t=2.11, df=16, p<0.05)

										(, •	, r	-)					
	Logistic					Monomolecular						Gompertz						
С	Polar diameter		Equatorial diameter		Polar diameter		Equatorial diameter			Polar diameter			Equatorial diameter					
	T1,T2	T1,T3	T2,T3	T1,T2	T1,T3	T2,T3	T1,T2	T1,T3	T2,T3	T1,T2	T1,T3	T2,T3	T1,T2	T1,T3	T2,T3	T1,T2	T1,T3	T2,T3
а	2.56	4.47	1.78	3.46	5.21	1.68	2.97	5.07	1.90	4.32	6.28	1.80	2.76	4.82	1.88	3.80	5.68	1.76
b	0.58	-0.36	-0.95	0.36	0.28	-0.12	0.53	-0.25	-0.74	0.47	0.34	-0.18	-0.55	0.32	0.86	-0.39	-0.29	0.14
с	-0.17	0.06	0.24	-0.14	-0.10	0.06	-0.14	0.05	0.18	-0.15	-0.10	0.06	0.15	-0.06	-0.21	0.14	0.09	-0.06
d	0.02	-0.01	-0.03	0.053	0.01	0.06	0.01	-0.00	-0.02	0.02	0.01	-0.01	-0.01	0.01	0.02	-0.02	-0.01	0.01
e	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03	-0.03	0.02	0.03	0.00	0.00	0.00	-0.00	0.00	0.00	0.00

T1: agronomic and pest management, T2: agronomic management, T3: without agronomic or pest management.

(Figure 2b). The results reported by Casierra-Posada et al. (2004) yielded values of 52, 34, 48 and 134, and the results reported by Lott (1942) yielded 56, 28, 39 and 123 days. Goodness of fit from a χ^2 test was calculated using the data from the present study as observed values and data from Lott (1942) and Casierra-Posada et al. (2004) as expected values. The analysis showed that χ^2 = 10.94, df=3, p=0.0042 and χ^2 = 9.95, df=2, p=0.006, respectively, indicating that there are significant differences between the data in this study and the data reported by Casierra-Posada et al. (2004) and Lott (1942). However, between these two authors there are no significant differences in growth periods. Despite this, the full blossoming dates from this study differed from Lott (1942) due to the differences in latitude, and also differed from Gutiérrez-Acosta et al. (2008) because of the difference in peach variety. The results of Gutiérrez-Acosta et al. (2008) are very similar to those of Donoso et al. (2008) as a consequence of the similarities between the first and the third stages.

Figure 2c shows a progressive decrease in the ratio of PD/ ED, implying an initial oblong shape in the first and second stage that becomes spherical in the third stage, and in coincidence with the data reported by Lott (1942) and Casierra-Posada et al. (2004). Even following the application of agronomical and pest control measures in T1 and agronomical measures in T2, there was no influence on time and growth rate but there was an influence on the fruit diameter.

The best fit among the double sigmoidal models in this study was the monomolecular model, according to largest values of R² and lowest SSE

and Akaike coefficients. The present study, therefore, is consistent with reports of the sweet orange, C. sinensis (Avanza et al., 2004), in which the MSE was used for model comparison and the Student's t test was used to test for differences between the coefficients. In addition, Rojas-Lara et al. (2008) found that the best fit for the 'Manzano' hot pepper (C. pubescens) data were the monomolecular model; for model comparisons, R^2 , χ^2 and MSE parameters were evaluated. However, Álvarez and Boche (1999) concluded that the best model for the simulation of late nectarine growth was the logistic model, and for the model evaluation they used R² and MSE. For the modeling of three tomato hybrids, Ardila et al. (2011) used the logistic model and only used MSE for comparisons. It is worth noting that Casierra-Posada et al. (2004) performed peach modeling, but they only employed third degree polynomials, which did not permit the comparison of final fruit diameter with the modeling estimates. Physical and physiological characteristics of the pear variety 'Triumph of Vienna' were measured by Parra-Coronado et al. (1998), but they did not model the data under the same conditions as in this study. Moreover, Donoso et al. (2008) conducted a peach fruit growth analysis without modeling. Even when fruit differs, the double sigmoid model is adequate to simulate fruit phenology, particularly the monomolecular model, and the statistics R^2 , χ^2 , t and MSE are useful for comparison.

Conclusion

The double sigmoidal models describe peach fruit growth with sufficient precision, and they are useful because they predict the appropriate timing for agronomical labor such as pruning, thinning, fertilization and insecticide application. The use of models based on thermal time accumulation is recommended, as is the measurement of additional characteristics such as the weight and the length of branches, as well as other climatic variables such as precipitation, humidity and solar radiation. These models could be successfully applied to the growth of other fruit from warm temperate climates.

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